BT-MRE-I&II-05.17-0670 Reg. No. [| | | | | | |]

B

B.Tech. Degree I & II Semester Examination in Marine Engineering May 2017

MRE 1101 ENGINEERING MATHEMATICS I

(2013 Scheme)

Time: 3 Hours Maximum Marks: 100

(5 × 20 = 100)

I. (a) Find the point where the following function is discontinuous. (4)

$$f(x)=\begin{cases} x & when\ 0\le x\le 1 \\ 2 & when\ x=1 \\ x+1 & when\ 1<x\le 2 \end{cases}.$$

(b) State Rolle's theorem and verify it for the function $f(x)=3x^4-4x^2+5$ in the interval $(-1,1)$ (6)

(c) Evaluate $\lim_{x\to 1}\dfrac{x^x-x}{x-1-\log x}$ (10)

OR

II. (a) Find the asymptotes of the curve $x^2y^2-x^2y-2xy^2+x+y+1=0$. (6)

(b) Find the n[th] derivative of $e^x(2x+3)^3$. (6)

(c) If $y=(\sin^{-1}x)^2$, show that $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-n^2y_n=0$. (8)

III. (a) Find the first and second order partial derivatives of $z=x^3+y^3-3axy$ and show that $\dfrac{\partial^2 z}{\partial y\partial x}=\dfrac{\partial^2 z}{\partial x\partial y}$. (6)

(b) If $u=\sin^{-1}\dfrac{x^2+y^2}{x+y}$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$. (8)

(c) The power P required to propel a steamer of length 'l' at a speed of u is given by $p=\lambda u^3 l^2$ where λ is a constant. If u is increased by 3% and l is decreased by 1%, find the corresponding increase in P. (6)

OR

IV. (a) If $u=\dfrac{yz}{x}$, $v=\dfrac{zx}{y}$, $w=\dfrac{xy}{z}$ compute $\dfrac{\partial(u,vw)}{\partial(x,yz)}$. (8)

(b) Find the minimum of $f(x,y)=x^2+y^2+6x+12$. (6)

(c) State Euler's theorem for homogeneous function and prove that if u is a homogeneous function in x and y of degree 'n', then (6)

$$x\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial^2 y}=(n-1)\frac{\partial u}{\partial y}.$$

(P.T.O.)

V. (a) Derive the equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1). (8)

(b) Find the centre, eccentricity, foci and direction of the hyperbola $9x^2-16y^2+18x+32y-151=0$ (6)

(c) Find the locus of the point of intersection of perpendicular normals of $y^2=4ax$. (6)

OR

VI. (a) Derive the equation of the normal at $\left(at_1^2, 2at_1\right)$ to the parabola $y^2=4ax$. (8)

(b) Find the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$ (6)

(c) Show that the line $lx+my+n=0$ is a normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)^2}{n^2}$. (6)

VII. (a) Find a reduction formula for $\int \sin^n x\,dx$ and hence evaluate $\int_0^a \frac{x^7}{\sqrt{\left(a^2-x^2\right)}}dx$. (10)

(b) Find the length of the arc of the parabola $x^2=4ay$ measured from the vertex to one extremity of latus rectum. (10)

OR

VIII. (a) Calculate by double integration the volume generated by the revolution of the cardioid $r=a(1-\cos\theta)$ about its axis. (10)

(b) Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$. (10)

IX. (a) Show that the vectors $\vec{a}-2\vec{b}-3\vec{c}, -2\vec{a}+3\vec{b}-4\vec{c}$ and $-\vec{b}+2\vec{c}$ are coplanar. (8)

(b) Given $\vec{a}=2\,\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=6\hat{i}-3\hat{j}+2\hat{k}$ find $\vec{a}X\vec{b}$ and a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. (6)

(c) Find grad ϕ where $\phi=3x^2y-y^3z^2$ at the point $(1, -2, -1)$. (6)

OR

X. (a) Find divergence and curl of the vector $\vec{v}=(xyz)\hat{i}+\left(3x^2y\right)\hat{j}+\left(xz^2-y^2z\right)\hat{k}$ at the point $(2, -1, 1)$. (5)

(b) A vector field is given $\vec{A}=\left(x^2+xy^2\right)\hat{i}+\left(y^2+x^2y\right)\hat{j}$. Show that the field is irrotational and find the scalar potential. (10)

(c) If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, show that $grad\ r=\frac{\vec{r}}{r}$. (5)

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